Stochastic

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Theorem: If E is a sample event whose sample point has n components $c_1, c_2, \dots, c_n$ each with probabilities $P(c_1), P(c_2), \dots, P(c_n)$, then

$$P(E) = P(c_1) \cdot P(c_2) \cdot P(c_3) \cdots P(c_n)$$

Example: From an experiment, it is known that 80% of all airplanes arriving in an airport are more than one hour late.
Randomly select and check the arrival time of 4 airplanes, recording "0" if a plane is less than one hour late, otherwise record L.

Solution

Sample space S = (0000), (000L), (00L0), (0L00), (L000), (00LL), (0L0L), (0LL0), (L0L0), (L00L), (LL00), (0LLL), (L0LL), (LL0L), (LLL0), (LLLL).

1 - what is the probability that the first 3 planes are less than one hour late?

2 - What is the prob. that the first plane is late and last plane is late?

⟹ turn over

$E1 = \{(OOOO), (OOOL)\}$, $E_2 = \{(LLLL), (LOLL), (LLOL), (LOOL)\}$

$P_L = 80\% = 0.8$

$P_O = 20\% = 0.2$

$P(E1) = P(OOOO) + P(OOOL)$

$= P(O).P(O).P(O).P(O) + P(O).P(O).P(O).P(L)$

$= (0.2)^4 + (0.2)^3 * (0.8) = \frac{1}{125}$

$P(E2) = P\{(LLLL), (LLOL), (LOLL), (LOOL)\}$

$= \left(\frac{8}{10}\right)^4 + \left(\frac{8}{10}\right)^3 \left(\frac{2}{10}\right) + \left(\frac{8}{10}\right)^3 \left(\frac{2}{10}\right) + \left(\frac{8}{10}\right)^2 \left(\frac{2}{10}\right)^2$

$= \frac{16}{25}$

Dril: What is the probability that both the first and third planes are less than one hour late?

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* Conditianal Probability:

$P(E|Z) = \frac{P(E \cap Z)}{P(Z)}$ احتمال حدوث E تحت شرط Z حيث
كل من E و Z عبارة عن Events

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

The conditional probability of an event A given that B has occured is denoted by $P(A|B)$ and defind by $P(A|B) = \frac{P(A \cap B)}{P(B)}$

→ turn over

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Example: Consider the experiment « survey 3 T.V. set owners to determine whether program "A" is being observed. Record "S" if the Answer is yes, "f" if the answer is no. Assume that the probability of the answer being yes = 1/2

Find $P(E_1|E_2)$, where E_1 the second person observes "A", E_2 the first person observes "A"

Solution

$E_1: \{(sss), (ssf), (fsf), (fsf)\}$

$E_2: \{(sss), (sfs), (ssf), (sff)\}$

Bay's Rule $\rightarrow$ $$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$(E_1 \cap E_2) = \{(sss), (ssf)\}$

$P(E_1 \cap E_2) = (1/2)^3 + (1/2)^3 = 1/4$

$P(E_2) = 1/8 + 1/8 + 1/8 + 1/8 = \frac{4}{8} = 1/2$

$P(E_1|E_2) = {}^{1/4}/_{1/2} = 1/2$

$\Rightarrow$ turn over

Example:

A manufacturer conducted a survey of 1000 TV viewers with the following results:

- Number of who saw his advertisement = 200
- 〃 〃 〃 〃 and purchased his product = 50
- Number of who didn't see the advertisement but purchased his product = 20

① What is the probability that a person who saw the advertisement was also a purchaser?

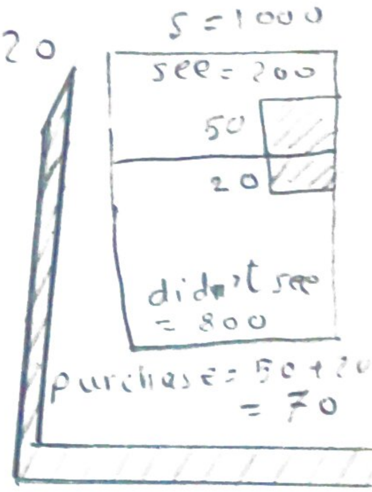
② What is the probability that a person who purchased also saw the advertisement?

$E_1 \Rightarrow$ person who purchased $= 50 + 20$

$E_2 \Rightarrow$ 〃 〃 saw $= 200$

$E_3 \Rightarrow$ 〃 〃 didn't see $= 800$

$S = 1000$



① $P(E_2 | E_1) = \dfrac{P(E_2 \cap E_1)}{P(E_1)}$

$(E_2 \cap E_1) = 50$

$P(E_2 | E_1) = \dfrac{50/1000}{70/1000} = 5/7$

② $P(E_1 | E_2) = \dfrac{50/1000}{200/1000} = \dfrac{5}{20} = \dfrac{1}{4}$

Independent events:

An event B is said to be independent of an event A if the probability that B occures is not influenced by whether A has occured or not. In other words, if the probability of B equals the conditional probability of B given A.

If $P(B) = P(B|A)$, A and B are independent,

but $P(B|A) = \frac{P(B \cap A)}{P(A)}$, then

$P(A) \cdot P(B) = P(B \cap A)$ if A and B are independent.

Example: the sample space for 3 childs' families is $S = \{(bbb, \qquad (ggg)\}$

if E_1: the first child is a boy.

E_2: " second " " girl.

E_3: two childs are boys.

E_4: last child is a girl

1- Are E_1 and E_2 independent?

2- Are E_3 and E_4 independent?

⟹ Turn over

Solution:

$E_1 = \{(bbb), (bbg), (bgb), (bgg)\}$

$E_2 = \{(bgb), (bgg)\ (ggb)\ (ggg)\}$

$E_3 = \{(bbg), (bbb), (bgb), (gbb)\}$

$E_4 = \{(ggg), (gbg), (bgg), (bbg)\}$

$(E_1 \cap E_2) = \{(bgb), (bgg)\}$

$P(E_1) = \frac{nE_1}{nS} = \frac{4}{8} = 1/2$

or $P(E_1) = (1/2 \cdot 1/2 \cdot 1/2) + (1/2 \cdot 1/2 \cdot 1/2) + \cdots + \cdots = \frac{4}{8} = 1/2$

$P(E_1 \cap E_2) = 2/8 = 1/4$, $P(E_1) \cdot P(E_2) = 1/2 \cdot 1/2 = 1/4$

Then, E_1 and E_2 are independent events.

$(E_3 \cap E_4) = (bbg)$

$P(E_3) = 1/2$, $P(E_4) = 1/2$

$P(E_3 \cap E_4) = 1/8$

but

$P(E_3) \cdot P(E_4) = 1/2 \cdot 1/2 = 1/4$

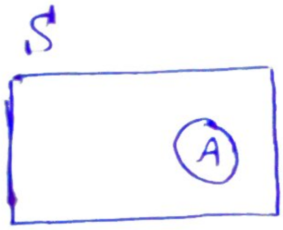
$\therefore P(E_3 \cap E_4) \neq P(E_3) \cdot P(E_4)$

Then, E_3 and E_4 are dependent events.

Theorem:

$$P(A^c) = 1 - P(A)$$

or $$P(A) = 1 - P(A^c)$$



If A and B are independent events, prove that

A^c and B^c are independent.

A^c and B 〃 〃 .

A and B^c 〃 〃 .